Pharmaceutical statistics

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Dr. Rola Aqeel

ARKAN ACADEMY

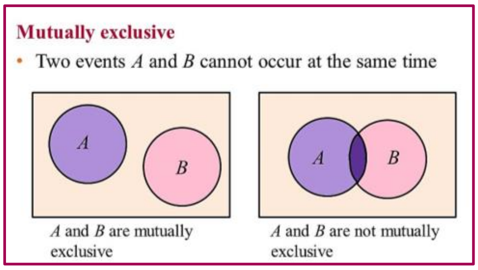
Probability

- ***Elementary properties of probability:***
  - Definition: Probability is the likelihood or ***chance*** of an event occurring.
  - It is expressed as a number between 0 and 1:
    - ✓ **0** indicates an impossible (null) event Φ.
    - ✓ **1** indicates a certain event Ω.
    - ✓ The rare event rule: if the probability of an event is low (less than 5%) then this event is unlikely to happen.
    - ✓ Any probability in between (e.g., 0.5) indicates a 50% chance of the event occurring.
    - ★ For example, when you flip a fair coin, the probability of getting heads is 0.5 (or 50%).
  - Probability of an event is usually referred to as ***relative frequency***.
  - Foundation of statistics because of the concept of sampling and the concept of variation and how likely an observed difference is due to chance (probability).

Number of Pets	Frequency	Relative Frequency	
1	150	37.5%	← 150/400 = 37.5%
2	90	22.5%	← 90/400 = 22.5%
3	110	27.5%	← 110/400 = 27.5%
4	30	7.5%	← 30/400 = 7.5%
5	20	5.0%	← 20/400 = 5.0%

  - Probability statements are used frequently in biostatistics.
    - ★ Examples:
      - We say that we are 90% sure that an observed treatment effect in a study is real.
      - The success probability of this surgery is only 10%
      - The probability of tumor developing is 50%.

- ***Properties of probability:***
  - Given some process (or experiment) with **n** mutually exclusive outcomes (called *events*), E1, E2, E3, …, En, the probability of any event E_i is assigned a *nonnegative* number. $P(E_i) \geq 0$
    - ✓ The outcomes are ***mutually exclusive***, meaning that if one outcome occurs, none of the others can occur simultaneously. For example, when rolling a die, getting a "1" and a "2" at the same time is impossible
    - ✓ In other words, <u>all events</u> must have a probability of more than or equal to zero (*no negative probability*)
  - The sum of probabilities of the mutually exclusive outcomes is equal to 1 (*exhaustiveness*):
    - ✓ $P(E1) + P(E2) + P(E3) + \ldots + P(En) = 1$
  - ***Mutual Exclusiveness:*** Two or more events are said to be mutually exclusive if the occurrence of any one of them means the others will not occur (That is, we cannot have 2 events occurring at the same time).
    - ★ Example:
      1. When rolling a die, getting a "1" and a "2" at the same time is impossible.
      2. Tossing a coin: the outcome of head or tail are a mutually exclusive event. No way in a certain toss you will have head and tail <u>at the same time</u>.
         It is either **H** or **T**
         S = {H, T}

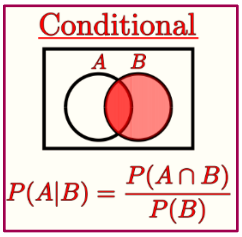


- ***Calculating the probability of an event***
  - Sample of 75 men and 36 women were selected to study the cocaine addiction as function of gender. The subjects are representative samples of typical adult who were neither in treatment nor in jail.

Frequency of cocaine use by gender among adult cocaine users			
Lifetime frequency of cocaine use	Male (M)	Female (F)	Total
1-19 times (A)	32	7	39
20-99 times (B)	18	20	38
100+ times (c)	25	9	34
Total	75	36	111

  - Suppose we pick a person at *random* from this sample, what is the probability that this person is a male?
    - ✓ 111 subjects are our population.
    - ✓ Male and female are mutually exclusive categories.
    - ✓ The likelihood of selecting any one person is equal to the likelihood of selecting any other
    - ✓ The desired probability is the number of subjects with the characteristic of interest (*Male*) divided by the total number of subjects.
    - ✓ ***P(M)*** = Number of males / Total number of subjects = 75/111 = 0.6757

- ***Conditional probability,* P (*B* | *A*)**



  - Conditional probability of an event B is the probability that the event will occur given the knowledge that an event A has already occurred. This probability is written ***P (B | A).***
  - $\boldsymbol{P\ (B \mid A) = \frac{P\ (B \cap A)}{P(A)}}$
  - Suppose we pick a subject at random from the 111 subjects and find that he is a male (M), what is the probability that this male will be one who has used cocaine 100 times or more during his lifetime?
    What is the probability that a subject has used cocaine 100 times or more given he is a male?
    - ✓ P (C_{100} | M) = 25/75 = ***0.33***

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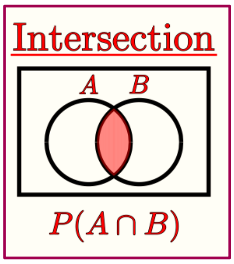
  - For mutually exclusive events ***P (B | A) = 0***

- ***Joint probability, P (A∩B)***



  - Joint probability is defined as the probability of both A and B taking place together (*at same time*)
  - The joint probability is given the symbolic notation: ***P (A∩ B)*** in which the symbol "∩" is read either as "intersection" or "and".
  - What is the probability that a person picked at random from the 111 subjects will be male ***and*** be a person who had used cocaine 100 times or more?
    - ✓ The statement $M \cap C_{100}$ indicates the joint occurrence of conditions M and C_{100}.
    - ✓ $P(M \cap C_{100})$ = 25/111 = 0.2252
    - ✓ The probability we seek is $P(M \cap C_{100})$.
    - ✓ We have already computed P(M)=75/111=0.6757 and a conditional probability P (C_{100} | M) =25/75=0.3333
    - ✓ We may now compute it by: $P(M \cap C_{100}) = P(M) * P(C_{100} \mid M)$ = 0.6757*0.3333=0.2252

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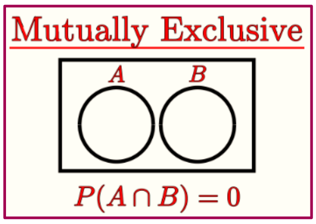
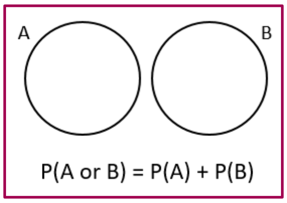


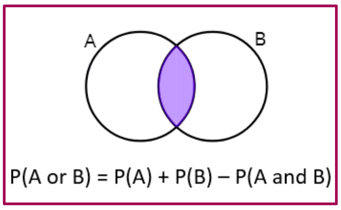
- When A and B *are mutually exclusive* variables, the probability of both occurring is zero:
  - ✓ ***P(A∩B) = 0***
- When A and B are *non-mutually exclusive* variables, the probability of both occurring is:
  - ✓ ***P(A∩B) = P(A)* P (B | A) or P(B)* P (A | B)***

- ***The Addition Rule, P (A U B) <u>for mutual exclusive events</u>***



  - In the previous example, if we picked a person at random from the 111 subject sample, what is the probability that this person will be a male (M) ***or*** female (F)?
  - We state this probability as P(M∪F) were the symbol ∪ is read either "union" or "or".
  - Since the two genders are mutually exclusive and variables:
  - ***P(M∪F) = P(M) + P(F)*** = (75/11) + (36/111) = 0.6757+0.3243=1
    - ★ Example:
    - ✓ If we <u>toss a dice</u>, we have 6 possibilities
    - ✓ S= {1, 2,3 4, 5, 6}. No way you will get any two "face up" at the same time.
    - ✓ We call these events: <u>Mutual exclusive events</u>.
    - ✓ Now:
    - ✓ What is the probability of having 3 faced up? P (3) =1/6 =0.1666
    - ✓ What is the probability of having 3 ***and*** 6 faced up together? P (3∩ 6) =0
      The events "getting a 3" and "getting a 6" are ***mutually exclusive*** because they cannot happen at the same time.
    - ✓ What is the probability of having 3 ***or*** 6 faced up? P (3∪6) =P (3) +P (6)=1/6+1/6=2/6=1/3

- ***The Addition Rule (OR, ∪) <u>for non-mutual exclusive e vents</u>***

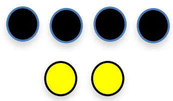


  - If the two events are not mutually exclusive:
  - Given two events A and B, the probability that event A or event B or both occur is equal to the probability that event A occurs, plus the probability that event B occurs minus the probability that events occur <u>simultaneously</u>:
  - ***P (A∪B) =P (A)+P (B)-P (A∩B)***
  - If we select a person at random from the 111 subjects, what is the probability that this person will be male (M) ***or*** will have used cocaine 100 times or more during his lifetime (C_{100}) or both?
    - ✓ $P(M \cup C_{100}) = P(M) + P(C_{100}) - P(M \cap C_{100})$
    - ✓ $P(M \cup C_{100}) = 0.6757 + 0.3063 - 0.2252 =$ **0.7568**
    - ✓ This result reflects the combined likelihood of either event occurring, considering that some individuals may fall into both categories (hence the subtraction of P(M∩C100).

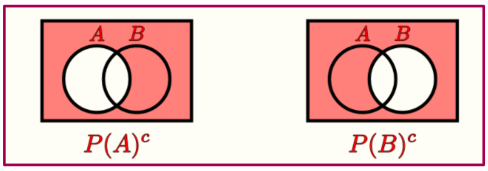
- ***Independent versus Dependent Events***
  - ***Independent***
    - ★ Example 1:
      - ✓ What is the probability of having a head (H) if we toss the first coin? ½
      - ✓ We did that and then we will toss the other coin:
      - ✓ What is the probability of having a head if we toss the <u>second coin</u> again? ½… so these events are called independent events.
      - ✓ "If the occurrence of event A *does not affect* the probability of event B to occur, then A&B are independent events"

    - ★ Example 2:
      - ✓ If we have 2 patients with 2 different Conditions
      - ✓ In patient A the probability of his surgery to success = 0.8
      - ✓ In patient B the probability of his surgery to success = 0.6
      - ✓ Q: what is the probability of A to success knowing that B failed? 0.8 (not affected by the result of B).
        - ***P (A | B) = P(A)*** (Because it's independent).
        - **P (A | B) =** $\frac{P(A \cap B)}{P(B)}$
        - **P (A) =** $\frac{P(A \cap B)}{P(B)}$
      - ✓ So, the P (A ∩ B)= P (A)*P (B) = 0.8*0.6= **0.48**
      - ✓ What is the probability that both or one of them will succeed?
        - ***P (A U B) = P (A) + P(B) – P (A ∩ B)= 0.8+0.6 - 0.48 = 0.92***

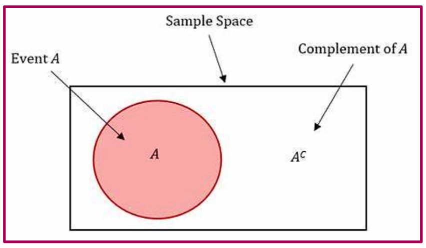
  - ***Dependent***
    - ★ Example:
    - ✓ All these balls in a bag, randomly we want to pick one.
    - ✓ What is the probability of picking a *yellow ball*: ***2/6=0.33***
    - ✓ If we did this really and the picked ball was yellow, what is the probability to pick a yellow ball if we pick one ***again***: ***1/5=0.2***
    - ✓ "If the occurrence of event A does ***affect*** the probability of event B to occur, then A&B are ***dependent*** events"



- ***Calculating the probability of an event***
  - Complimentary events:
    - ✓ The probability of an event A is equal to 1 minus the probability of its compliment which is written as Ā.
    - ✓ ***Rules of Complimentary events***
      - ***P (Ā) P(A) = 1***
      - ***P (Ā) = 1-P (A)***
      - ***P (A) = 1- P (Ā)***



    - ★ Example:
    - ✓ If the probability of having smokers in this class is 5%, so what is the probability of having non-smokers in the same class? P(Non-Smokers) =1−P (Smokers)=1−0.05=0.95 or 95%
    - ✓ If you toss a dice, what is the probability not to have 2 "face up"? P (Not getting a 2) =1−P (Getting a 2) =1−1/6=5/6



- ***Questions for practice:***

1) Cards, each labeled with a number from 0 to 9, one card is picked randomly.
What is the probability that the number on the card is:

(a) ***odd:***

The odd numbers in the set are: {1, 3, 5, 7, 9}.
There are 5 odd numbers out of 10 possible outcomes.
P(Odd) =5/10=1/2=0.5 or 50%

(b) ***a multiple of 3***:

The multiples of 3 in the set are: {3, 6, 9}.
There are 3 multiples of 3 out of 10 possible outcomes.
P (Multiple of 3) =310=0.3 or 30%

(c) ***a 5***:

P (5) =1/10 = 0.1 or 10%

(d) ***not a 7*****:**

The numbers that are not 7 are: {0, 1, 2, 3, 4, 5, 6, 8, 9}.
There are 9 numbers that are not 7 out of 10 possible outcomes.
P (Not a 7) = 9/10=0.9 or 90%

2) If we toss a coin ***three times*****:**

a) What is the probability of having two heads and one tail?
All events: {HHH, HHT, HTH, HTT, TTT, TTH, THT, THH}
P (HHT or HTH or THH) = P (HHT)+P(HTH)+P(THH)
=1/2*1/2*1/2 + 1/2*1/2*1/2 + 1/2*1/2*1/2 = ***3/8*** = ***0.375***

b) Now if the coin is biased so that the head is twice as likely to occur as a tail, what is the probability to have two heads and one tail?
If the coin is biased such that the probability of getting heads is twice the probability of getting tails, let p be the probability of getting tails.

- Then the probability of getting heads is P(H) = 2p and P (T)= p
- Since the total probability must add up to 1:
  P(H)+P(T) = 2p + p =3p =1 ⟹ p =1/3
- Therefor: ***P(H) =2p =2×1/3 =2/3*** and ***P(T)=1/3***

✓ P (HHT or HTH or THH) = P (HHT)+P(HTH)+P(THH)
= 2/3*2/3*1/3 + 2/3*1/3*2/3 + 1/3*2/3*2/3 = ***4/9 = 0.44***

3) Questions on this table:

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1. P (M)
2. P (C)
3. P (M ∩ B)
4. P (A | F)
5. P (F | B)
6. P (A ∩ B)
7. P (A U B)
8. What is the probability to select a female using cocaine 1-19 times? P (F ∩ A)
9. If we found out that the selected person is a woman, what is the probability to be using cocaine 1-19 times? P (A | F)
10. What is the probability for a selected subject here to be male and have used cocaine 1-19 times?
11. What is the probability for a person selected randomly to be male or female? **1**
12. What is the probability for a selected subject to be male ***or*** have used cocaine 1-19 times or both?
    P (M U A) = P (M) + P (A) – P (M ∩ A)

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+962 790408805